Multi-Neighborhood Simulated Annealing for the Sports Timetabling Competition ITC2021

Roberto Maria Rosati[1], Matteo Petris[2], Luca di Gaspero[1], Andrea Schaerf[1]

(1) University of Udine, Italy

(2) Univ. Lille, Inria, CNRS, France

Outline

Definition

- 5th International Timetabling Competition (ITC2021)
  - 1st edition about Sports Timetabling
  - Organized by David Van Bulck, Dries Goossens, Jeroen Beliën, Morteza Davari

Multi-Neighborhood Simulated Annealing ranked at

2nd place

at ITC2021

Problem Illustration

- Double round-robin timetable, time-constrained (compact)
- Nine constraint types from five different classes
  - They can be *hard* (feasibility) or *soft* (objective)
- Instances can be *phased* or *not phased* (never mirrored).

round_0	round_1	round_2	round_3	round_4
2 - 0	4 - 0	3 - 0	0 - 5	0 - 1
1 - 3	2 - 1	5 - 1	1 - 4	5 - 2
4 - 5	3 - 5	4 - 2	3 - 2	3 - 4
round_5	round_6	round_7	round_8	round_9
0 - 3	0 - 4	0 - 2	5 - 0	1 - 0
1 - 5	1 - 2	3 - 1	4 - 1	2 - 5
2 - 4	3 - 5	5 - 4	2 - 3	4 - 3

Constraints

- **Capacity (CA1, CA2, CA3, CA4)**: Force a team to play home or away and regulate the total number of games played by a team or group of teams.
- **Games (GA1)**: Enforce or forbid specific assignments of a game to time slots.
- **Breaks (BR1, BR2)**: Regulate the frequency and timing of breaks in a competition.
- **Fairness (FA2)**: Increase the fairness and attractiveness of competitions.
- **Separation (SE1)**: regulate the number of rounds between consecutive games involving the same teams.

Phased vs Not Phased timetable

Phased

round_0	round_1	round_2	round_3	round_4
2 - 0	4 - 0	3 - 0	0 - 5	0 - 1
1 - 3	2 - 1	5 - 1	1 - 4	5 - 2
4 - 5	3 - 5	4 - 2	3 - 2	3 - 4

round_5	round_6	round_7	round_8	round_9
0 - 4	0 - 3	0 - 2	5 - 0	1 - 0
1 - 2	1 - 5	3 - 1	4 - 1	2 - 5
3 - 5	2 - 4	5 - 4	2 - 3	4 - 3

Not Phased

round_0	round_1	round_2	round_3	round_4
2 - 0	4 - 0	3 - 0	0 - 3	0 - 2
1 - 3	2 - 1	5 - 1	1 - 5	3 - 1
4 - 5	3 - 5	4 - 2	2 - 4	5 - 4

round_5	round_6	round_7	round_8	round_9
0 - 5	0 - 4	5 - 0	1 - 0	0 - 1
1 - 4	1 - 2	4 - 1	2 - 5	5 - 2
3 - 2	3 - 5	2 - 3	4 - 3	3 - 4

Related Work

- *Gelling (1973)*: Relationship between 1-factorizations of a complete graph and the Sports Timetabling Problem.
- *Russell (1980)*: Carry-over effect
- *de Werra (1981)*: Canonical pattern
- *Wallis (1983)*: Home-away pattern

Related Work

- Metaheuristics:
  - *Costa (1995)*: Evolutionary Tabu Search
  - *Della Croce, Tadei, and Asioli (1995)*: Tabu Search
  - *Hamiez and Hao (2000)*: Tabu Search
  - Traveling Tournament Problem (*Easton, Nemhauser, Trick, 2001*)
    - *Ribeiro and Urrutia (2004)*: GRASP, Iterated Local Search
    - *Anagnostopoulos, Michel, Van Hentenryck, and Vergados (2006)*: Simulated Annealing
    - *Di Gaspero and Schaerf (2007)*: Tabu Search

Related Work

- *Thompson (2011)*, *Costa, Urrutia, and Ribeiro (2012)*, *Juanuario and Urrutia (2016)*: Further heuristics and neighborhoods
- *Van Bulck, Goossens, Schönberger, Guajardo (2021)*: RobinX
- Surveys:
  - *Rasmussen and Trick (2008)*
  - *Kendall, Knust, Ribeiro, and Urrutia (2010)*

Simulated Annealing - Motivation

- Consolidated state-of-the-art local search operators
- Good results on similar problems (e.g.: Traveling Tournament Problem)
- Our MIP model solved by CPLEX did not yield good results in reasonable time

Search Space

$round_0$	$round_1$	$round_2$	$round_3$	$round_4$
2 - 0	4 - 0	3 - 0	0 - 5	0 - 1
1 - 3	2 - 1	5 - 1	1 - 4	5 - 2
4 - 5	3 - 5	4 - 2	3 - 2	3 - 4

$round_5$	$round_6$	$round_7$	$round_8$	$round_9$
0 - 3	0 - 4	0 - 2	5 - 0	1 - 0
1 - 5	1 - 2	3 - 1	4 - 1	2 - 5
2 - 4	3 - 5	5 - 4	2 - 3	4 - 3

	$round_0$	$round_1$	$round_2$	$round_3$	$round_4$	$round_5$	$round_6$	$round_7$	$round_8$	$round_9$
$team_0$	-2	-4	-3	+5	+1	+3	+4	+2	-5	-1
$team_1$	+3	-2	-5	+4	-0	+5	+2	-3	-4	+0
$team_2$	+0	+1	-4	-3	-5	+4	-1	-0	+3	+5
$team_3$	-1	-5	+0	+2	+4	-0	+5	+1	-2	-4
$team_4$	+5	+0	+2	-1	-3	-2	-0	-5	+1	+3
$team_5$	-4	+3	+1	-0	+2	-1	-3	+4	+0	-2

Initial Solution

- Random
  1. Single round-robin canonical pattern
  2. Random team permutation
  3. Mirror
  4. Random rounds permutation
- Greedy
  - At every iterations, the candidate that violates the least number of hard constraints is added to the solution
  - Based on the canonical pattern

Simulated Annealing - Neighborhoods

SwapHomes	$\mathcal{SH}\langle t_i, t_j\rangle$
SwapTeams	$\mathcal{ST}\langle t_i, t_j\rangle$
SwapRounds	$\mathcal{SR}\langle r_i, r_j\rangle$
PartialSwapTeams	$\mathcal{PST}\langle t_i, t_j, \mathcal{R}_s\rangle$
PartialSwapRounds	$\mathcal{PSR}\langle \mathcal{T}_s, r_i, r_j\rangle$
PartialSwapTeamsPhased	$\mathcal{PSTP}\langle t_i, t_j, \mathcal{R}_s\rangle$

Move $\mathcal{SH}\langle t_i, t_j \rangle$

Attributes $t_i, t_j \in \mathcal{T},\ t_i \neq t_j$

Effect swaps the home/away position of the two games between t_i and t_j

-2	-4	-3	+5	+1	+3	+4	+2	-5	-1
+3	-2	-5	+4	-0	+5	+2	-3	-4	+0
+0	+1	-4	-3	-5	+4	-1	-0	+3	+5
-1	-5	+0	+2	**-4**	-0	+5	+1	-2	**+4**
+5	+0	+2	-1	**+3**	-2	-0	-5	+1	**-3**
-4	+3	+1	-0	+2	-1	-3	+4	+0	-2

$\longrightarrow$

-2	-4	-3	+5	+1	+3	+4	+2	-5	-1
+3	-2	-5	+4	-0	+5	+2	-3	-4	+0
+0	+1	-4	-3	-5	+4	-1	-0	+3	+5
-1	-5	+0	+2	**+4**	-0	+5	+1	-2	**-4**
+5	+0	+2	-1	**-3**	-2	-0	-5	+1	**+3**
-4	+3	+1	-0	+2	-1	-3	+4	+0	-2

$\mathcal{SH}\langle 3, 4 \rangle$

SwapTeams

Move $\mathcal{ST}\langle t_i, t_j \rangle$

Attributes $t_i, t_j \in \mathcal{T},\ t_i \neq t_j$

Effect swaps the positions of t_i and t_j throughout the whole timetable.

-3	**-2**	**+4**	**-4**	**-1**	**+2**	**+1**	**+5**	**-5**	**+3**
+4	-3	-5	+5	+0	+3	-0	+2	-2	-4
+5	+0	+3	-3	-4	-0	+4	-1	+1	-5
+0	+1	-2	+2	+5	-1	-5	-4	+4	-0
-1	**-5**	**-0**	**+0**	**+2**	**+5**	**-2**	**+3**	**-3**	**+1**
-2	+4	+1	-1	-3	-4	+3	-0	+0	+2

$\longrightarrow$

-1	**-5**	**-0**	**+0**	**+2**	**+5**	**-2**	**+3**	**-3**	**+1**
+0	-3	-5	+5	+4	+3	-4	+2	-2	-0
+5	+4	+3	-3	-0	-4	+0	-1	+1	-5
+4	+1	-2	+2	+5	-1	-5	-0	+0	-4
-3	**-2**	**+4**	**-4**	**-1**	**+2**	**+1**	**+5**	**-5**	**+3**
-2	+0	+1	-1	-3	-0	+3	-4	+4	+2

$\mathcal{ST}\langle 0, 4 \rangle$

SwapRounds

Move $\mathcal{SR}\langle r_i, r_j \rangle$

Attributes $r_i, r_j \in \mathcal{R},\ r_i \neq r_j$

Effect swaps the two rounds in the timetable.

-2	-4	**-3**	+5	+1	+3	+4	+2	**-5**	-1
+3	-2	**-5**	+4	-0	+5	+2	-3	**-4**	+0
+0	+1	**-4**	-3	-5	+4	-1	-0	**+3**	+5
-1	-5	**+0**	+2	+4	-0	+5	+1	**-2**	-4
+5	+0	**+2**	-1	-3	-2	-0	-5	**+1**	+3
-4	+3	**+1**	-0	+2	-1	-3	+4	**+0**	-2

$\longrightarrow$

-2	-4	**-5**	+5	+1	+3	+4	+2	**-3**	-1
+3	-2	**-4**	+4	-0	+5	+2	-3	**-5**	+0
+0	+1	**+3**	-3	-5	+4	-1	-0	**-4**	+5
-1	-5	**-2**	+2	+4	-0	+5	+1	**+0**	-4
+5	+0	**+1**	-1	-3	-2	-0	-5	**+2**	+3
-4	+3	**+0**	-0	+2	-1	-3	+4	**+1**	-2

$\mathcal{SR}\langle 2, 8 \rangle$

PartialSwapTeams

Move $\mathcal{PST}\langle t_i, t_j, \mathcal{R}_s\rangle$

Attributes two teams $t_i, t_j \in \mathcal{T}$, $t_i \neq t_j$, a set of rounds $\mathcal{R}_s = \{r_1, \dots, r_s\}$, $\mathcal{R}_s \subset \mathcal{R}$

Effect swaps the positions of t_i and t_j on the set of rounds in $\mathcal{R}_s$

- $\mathcal{R}_s$ must be built so that the double round-robin structure is conserved

-3	+5	**-5**	**-2**	-1	+2	**+1**	**+3**	+4	-4
-5	+3	+2	-4	+0	+4	-0	-2	+5	-3
+4	-4	-1	+0	-3	-0	-5	+1	+3	+5
+0	-1	+4	+5	+2	-5	-4	-0	-2	+1
-2	+2	**-3**	**+1**	+5	-1	**+3**	**-5**	-0	+0
+1	-0	+0	-3	-4	+3	+2	+4	-1	-2

$\longrightarrow$

-2	+5	**-3**	**+1**	-1	+2	**+3**	**-5**	+4	-4
-5	+3	+2	-0	+0	+4	-4	-2	+5	-3
+0	-4	-1	+4	-3	-0	-5	+1	+3	+5
+4	-1	+0	+5	+2	-5	-0	-4	-2	+1
-3	+2	**-5**	**-2**	+5	-1	**+1**	**+3**	-0	+0
+1	-0	+4	-3	-4	+3	+2	+0	-1	-2

$\mathcal{PST}\langle 0, 4, \{0, 2, 3, 6, 7\}\rangle$

PartialSwapRounds

Move $\mathcal{PSR}\langle \mathcal{T}_s, r_i, r_j \rangle$

Attributes two rounds $r_i, r_j \in \mathcal{R}$, $r_i \neq r_j$, and a set of teams $\mathcal{T}_s = \{t_1, \ldots, t_s\}$, $\mathcal{T}_s \subset \mathcal{T}$

Effect swaps the matches including teams in $\mathcal{T}_s$ between rounds r_i and r_j.

- $\mathcal{T}_s$ must be built so that the double round-robin structure is conserved

-2	-4	+3	-1	+5	-3	+1	+2	-5	+4
+4	-3	-5	+0	+2	-2	-0	**+5**	-4	+3
+0	+5	-4	+3	-1	+1	+4	-0	-3	-5
+5	+1	-0	-2	+4	+0	-5	**-4**	+2	-1
-1	+0	+2	-5	-3	+5	-2	**+3**	+1	-0
-3	-2	+1	+4	-0	-4	+3	**-1**	+0	+2

$\longrightarrow$

-2	-4	+3	-1	+5	-3	+1	+2	-5	+4
+5	-3	-5	+0	+2	-2	-0	**+4**	-4	+3
+0	+5	-4	+3	-1	+1	+4	-0	-3	-5
-4	+1	-0	-2	+4	+0	-5	**+5**	+2	-1
+3	+0	+2	-5	-3	+5	-2	**-1**	+1	-0
-1	-2	+1	+4	-0	-4	+3	**-3**	+0	+2

$\mathcal{PSR}\langle \{1, 3, 4, 5\}, 0, 7 \rangle$

PartialSwapTeamsPhased

- Inspired by PartialSwapTeams
- Limits the disruptive (side) effects of PartialSwapTeams on phased instances
- Based on the concept of mixed phase and mixed leg
- It does not modify the current phase state

Mixed Phase

-2	-4	-3	+5	+1	+3	+4	+2	-5	-1
+3	-2	-5	+4	-0	+5	+2	-3	-4	+0
+0	+1	-4	-3	-5	+4	-1	-0	+3	+5
-1	-5	+0	+2	+4	-0	+5	+1	-2	-4
+5	+0	+2	-1	-3	-2	-0	-5	+1	+3
-4	+3	+1	-0	+2	-1	-3	+4	+0	-2

↔

0	0	0	0	0	1	1	1	1	1
0	0	0	0	0	1	1	1	1	1
0	0	0	0	0	1	1	1	1	1
0	0	0	0	0	1	1	1	1	1
0	0	0	0	0	1	1	1	1	1
0	0	0	0	0	1	1	1	1	1

-4	-5	+3	+4	-3	+2	+1	-1	-2	+5
-2	-3	+2	+5	-5	+3	-0	+0	-4	+4
+1	-4	-1	-3	+4	-0	-5	+5	+0	+3
-5	+1	-0	+2	+0	-1	+4	-4	+5	-2
+0	+2	+5	-0	-2	-5	-3	+3	+1	-1
+3	+0	-4	-1	+1	+4	+2	-2	-3	-0

↔

0	0	0	1	1	0	0	1	1	1
0	0	1	0	1	1	0	1	0	1
0	0	1	0	1	0	0	1	1	1
0	0	0	0	1	1	0	1	1	1
0	0	0	1	1	1	0	1	0	1
0	0	0	0	1	1	0	1	1	1

PartialSwapTeamsPhased

Move $\mathcal{PSTP}\langle t_i, t_j, \mathcal{R}_s \rangle$

Attributes two teams $t_i, t_j \in \mathcal{T}$, $t_i \neq t_j$, and a set of rounds $\mathcal{R}_s = \{r_1, \ldots, r_s\}$, $\mathcal{R}_s \subset \mathcal{R}$.

Effect swaps of the positions of t_i and t_j on the set of rounds in $\mathcal{R}_s$.

- $\mathcal{T}_s$ must be built so that the double round-robin structure is conserved
- matches involved in the move must belong to the same mixed leg
- t_i and t_j keep their h/a positions against same opponent

PSTP: Example 1

$\mathcal{PSTP}\langle 2, 8, \{0, 1, 3, 4, 5, 8\}\rangle$

-5	+7	+8	-3	-9	+1	+2	-6	+4	-1	+3	+5	+9	-4	-8	-7	+6	-2
-7	+6	+5	-8	-2	-0	+3	-4	+9	+0	+8	+7	+2	-9	-5	-6	+4	-3
-4	**+9**	+6	**+5**	**+1**	**-7**	-0	-8	**-3**	+7	-5	+4	-1	+3	-6	-9	+8	+0
-6	+4	+7	+0	-8	-5	-1	-9	+2	+5	-0	+6	+8	-2	-7	-4	+9	+1
+2	-3	-9	+6	+5	+8	-7	+1	-0	-8	-6	-2	-5	+0	+9	+3	-1	+7
+0	+8	-1	-2	-4	+3	+9	-7	+6	-3	+2	-0	+4	-6	+1	-8	+7	-9
+3	-1	-2	-4	+7	+9	+8	+0	-5	-9	+4	-3	-7	+5	+2	+1	-0	-8
+1	-0	-3	-9	-6	+2	+4	+5	+8	-2	+9	-1	+6	-8	+3	+0	-5	-4
+9	**-5**	-0	**+1**	**+3**	**-4**	-6	+2	**-7**	+4	-1	-9	-3	+7	+0	+5	-2	+6
-8	-2	+4	+7	+0	-6	-5	+3	-1	+6	-7	+8	-0	+1	-4	+2	-3	+5

$\rightarrow$

-5	+7	+8	-3	-9	+1	+2	-6	+4	-1	+3	+5	+9	-4	-8	-7	+6	-2
-7	+6	+5	-2	-8	-0	+3	-4	+9	+0	+8	+7	+2	-9	-5	-6	+4	-3
+9	**+5**	+6	**+1**	**-3**	**-4**	-0	-8	**-7**	+7	-5	+4	-1	+3	-6	-9	+8	+0
-6	+4	+7	+0	+2	-5	-1	-9	-8	+5	-0	+6	+8	-2	-7	-4	+9	+1
+8	-3	-9	+6	+5	+2	-7	+1	-0	-8	-6	-2	-5	+0	+9	+3	-1	+7
+0	-2	-1	+8	-4	+3	+9	-7	+6	-3	+2	-0	+4	-6	+1	-8	+7	-9
+3	-1	-2	-4	+7	+9	+8	+0	-5	-9	+4	-3	-7	+5	+2	+1	-0	-8
+1	-0	-3	-9	-6	+8	+4	+5	+2	-2	+9	-1	+6	-8	+3	+0	-5	-4
-4	**+9**	-0	**-5**	**+1**	**-7**	-6	+2	**+3**	+4	-1	-9	-3	+7	+0	+5	-2	+6
-2	-8	+4	+7	+0	-6	-5	+3	-1	+6	-7	+8	-0	+1	-4	+2	-3	+5

0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1

$\leftrightarrow$

0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1

PSTP: Example 2

$\mathcal{PSTP}\langle 2, 8, \{10, 11, 12, 13, 14, 15, 17\}\rangle$

+2	-8	-9	+5	+9	-7	+7	-6	+1	+3	-4	-3	+8	-5	+4	-2	-1	+6
+7	-3	+5	+6	-6	-8	+3	+2	-0	-9	+9	+4	-2	-7	+8	-4	+0	-5
-0	-9	+3	+4	+8	+5	-4	-1	-7	-6	**-5**	**+9**	**+1**	**+6**	**+7**	**+0**	-8	**-3**
+4	+1	-2	+8	+5	+9	-1	-5	-4	-0	-7	+0	+7	-8	-9	-6	+6	+2
-3	-5	+6	-2	+7	-6	+2	-8	+3	+8	+0	-1	+5	+9	-0	+1	-7	-9
-8	+4	-1	-0	-3	-2	+6	+3	-9	+7	+2	+8	-4	+0	-6	-7	+9	+1
-9	-7	-4	-1	+1	+4	-5	+0	+8	+2	-8	+7	+9	-2	+5	+3	-3	-0
-1	+6	+8	+9	-4	+0	-0	-9	+2	-5	+3	-6	-3	+1	-2	+5	+4	-8
+5	+0	-7	-3	-2	+1	+9	+4	-6	-4	**+6**	**-5**	**-0**	**+3**	**-1**	**-9**	+2	**+7**
+6	+2	+0	-7	-0	-3	-8	+7	+5	+1	-1	-2	-6	-4	+3	+8	-5	+4

→

+2	-8	-9	+5	+9	-7	+7	-6	+1	+3	-4	-3	-2	-5	+4	+8	-1	+6
+7	-3	+5	+6	-6	-8	+3	+2	-0	-9	+9	+4	+8	-7	-2	-4	+0	-5
-0	-9	+3	+4	+8	+5	-4	-1	-7	-6	**+6**	**-5**	**+0**	**-3**	**+1**	**+9**	-8	**+7**
+4	+1	-2	+8	+5	+9	-1	-5	-4	-0	-7	+0	+7	+2	-9	-6	+6	-8
-3	-5	+6	-2	+7	-6	+2	-8	+3	+8	+0	-1	+5	+9	-0	+1	-7	-9
-8	+4	-1	-0	-3	-2	+6	+3	-9	+7	+8	+2	-4	+0	-6	-7	+9	+1
-9	-7	-4	-1	+1	+4	-5	+0	+8	+2	-2	+7	+9	-8	+5	+3	-3	-0
-1	+6	+8	+9	-4	+0	-0	-9	+2	-5	+3	-6	-3	+1	-8	+5	+4	-2
+5	+0	-7	-3	-2	+1	+9	+4	-6	-4	**-5**	**-9**	**-1**	**+6**	**+7**	**-0**	+2	**+3**
+6	+2	+0	-7	-0	-3	-8	+7	+5	+1	-1	+8	-6	-4	+3	-2	-5	+4

0	0	0	0	1	0	1	0	0	0	0	1	1	1	1	1	1	1
0	0	0	0	1	0	1	0	0	0	1	0	1	1	1	1	1	1
0	0	0	0	0	0	1	0	0	0	1	1	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	0	0	1	1	1	1	0	1	1
0	0	0	0	0	1	1	0	1	1	0	0	1	0	1	1	1	1
0	0	0	0	0	0	0	1	0	0	1	1	1	1	1	1	1	1
0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	1	1
0	0	0	0	0	0	1	1	0	0	0	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	1	0	0	1	0	0	1	1	1	0	1	1	1	1

↔

0	0	0	0	1	0	1	0	0	0	0	1	1	1	1	1	1	1
0	0	0	0	1	0	1	0	0	0	1	0	1	1	1	1	1	1
0	0	0	0	0	0	1	0	0	0	1	1	1	1	1	1	1	1
0	0	0	0	0	0	1	1	1	0	0	1	1	1	1	0	1	1
0	0	0	0	0	1	1	0	1	1	0	0	1	0	1	1	1	1
0	0	0	0	0	0	0	1	0	0	1	1	1	1	1	1	1	1
0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	1	1
0	0	0	0	0	0	1	1	0	0	0	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	1	0	0	1	0	0	1	1	1	0	1	1	1	1

Multi-Neighborhood

Union of individual neighborhoods

$$MN = SH \cup ST \cup SR \cup PST \cup PSR \cup PSTP$$

Selection criterion: random (suitable for Simulated Annealing)

- Each neighborhood is assigned with a probability
  - $p(SH) + p(ST) + p(SR) + p(PST) + p(PSR) + p(PSTP) = 1$
- Selections steps:
  1. Select at random a neighborhood, according to their probability
  2. Select uniformly a move inside the chosen neighborhood

Simulated Annealing

procedure *SimulatedAnnealing*(SearchSpace $\mathcal{S}$, Neighborhood $\mathcal{N}$, CostFunction F, Parameters T_0, T_f, α, N_s)

$T \leftarrow T_0$
$s \leftarrow \mathit{RandomState}(\mathcal{S})$
$s_{best} \leftarrow s$
while $T \geq T_f$
$n_s \leftarrow 0$
while $n_s < N_s$
$m \leftarrow \mathit{RandomMove}(s, \mathcal{N})$
$\Delta F \leftarrow F(s \oplus m) - F(s)$
if $(\Delta F \leq 0)$
$s \leftarrow s \oplus m$
if $(F(s) < F(s_{best}))$
$s_{best} \leftarrow s$
else
if $(\mathit{RandomReal}(0,1) < e^{-\Delta F / T})$
$s \leftarrow s \oplus m$
$n_s \leftarrow n_s + 1$
$T \leftarrow T \cdot \alpha$
return s_{best}

Simulated Annealing

procedure *SimulatedAnnealing*(SearchSpace $\mathcal{S}$, Neighborhood $\mathcal{N}$, CostFunction F, Parameters T_0, T_f, α, N_s, N_a)

```
T ← T_0
s ← RandomState(S)
s_best ← s
while T ≥ T_f
    n_s ← 0
    n_a ← 0
    while n_s < N_s ∧ n_a < N_a
        m ← RandomMove(s, N)
        ΔF ← F(s ⊕ m) − F(s)
        if (ΔF ≤ 0)
            s ← s ⊕ m
            n_a ← n_a + 1
            if (F(s) < F(s_best))
                s_best ← s
        else
            if (RandomReal(0, 1) < e^(−ΔF/T))
                s ← s ⊕ m
                n_a ← n_a + 1
        n_s ← n_s + 1
    T ← T · α
return s_best
```

Three-Stages

Stage 1: Find a feasible solution

Stage 2: Optimize

Stage 3: Move to nearest local minimum

	Constraints		Search space
	Hard	Soft	regions explored
Stage 1	Weighted objective	Not active	Feasible & infeasible
Stage 2	Weighted objective	Objective	Feasible & infeasible
Stage 3	Sctrictly hard	Objective	Only feasible

Instances

- 45 instances from ITC2021 competition
  - 16,18,20 teams, phased/not phased
  - 51 to 246 hard constraints, 34 to 1231 soft constraints
  - Hardest: `Middle_2` (phased, 16 teams, 246 H, 1231 S)
- instances, validator and benchmarks available on competition website: https://www.sportscheduling.ugent.be/ITC2021/

Size	¬ Phased	Phased	Total
16	4	5	9
18	8	10	18
20	11	7	18
Total	23	22	45

Parameters Tuning

- Tuned parameters
  - Neighborhoods probabilities for phased and not-phased instances
  - Simulated Annealing parameters for the three stages
  - Hard constraint weights for stages 1 and 2
- `json2run` [Urli, 2013] (F-Race, Hammersley, Friedman and Wilcoxon)

Parameter	Description	Tuning Range	Assigned Values		
			Not Phased		Phased
p_{sh}	SwapHomes	[0.0, 1.0]	0.154		0.130
p_{st}	SwapTeams	[0.0, 1.0]	0.070		0.020
p_{sr}	SwapRounds	[0.0, 1.0]	0.025		0.080
p_{pst}	PartialSwapTeams	[0.0, 1.0]	0.319		0.120
p_{pstp}	PartialSwapTeamsPhased	[0.0, 1.0]	0.070		0.130
p_{psr}	PartialSwapRounds	[0.0, 1.0]	0.350		0.520
			Stage 1	Stage 2	Stage 3
T_0	Start Temperature	[0, 2000]	179	600	17.9
T_{min}	End Temperature	[0, 20]	2.1	3.52	0.21
w_{h,ca_1}	Weight of CA1 hard	[1, 10]	7	7	-
w_{h,ca_2}	Weight of CA2 hard	[1, 10]	8	8	-
w_{h,ca_3}	Weight of CA3 hard	[1, 10]	2	2	-
w_{h,ca_4}	Weight of CA4 hard	[1, 10]	8	8	-
w_{h,ga_1}	Weight of GA1 hard	[1, 10]	10	10	-
w_{h,br_1}	Weight of BR1 hard	[1, 10]	1	1	-
w_{h,br_2}	Weight of BR2 hard	[1, 10]	6	6	-
w_{h,fa_2}	Weight of FA2 hard	[1, 10]	1	1	-
w_{h,se_1}	Weight of SE1 hard	[1, 10]	1	1	-
	Initial solution	{*random*, *greedy*}	*greedy*	-	-
w_h	Feature-dependent w_h	{*yes*, *no*}	*no*	*yes*	-
w_p	Feature-dependent w_p	{*yes*, *no*}	*no*	*yes*	-
k	Correlation factor	[0.1, 1000.0]	-	0.5	-
w_h	Hard weight (fixed)	[1, 1000]	10	-	-
w_p	Phased weight (ixed)	[1, 1000]	117	-	-

Experimental Set

- Fixed number of iterations per stage (0.6 - 13 hours depending on the instance)
- C++, Ubuntu 20.04.2 LTS
- AMD Ryzen Threadripper PRO 3975WX processor with 32 cores, hyper-threaded to 64 virtual cores, with base clock frequency of 3.5 GHz, and 64 GB of RAM
- 1 CPU per run.

Solutions

Inst.	Best	Average values cost	time (*h*)	feas.	Best	CPLEX LB
E1	423	540.7	1,6	1.00	362	1.0
E2	318	384.6	4,1	1.00	145	0.0
E3	1068	1176.5	3,4	1.00	992	48.9
E4	556	1007.8	2,4	0.56	507	0.0
E5	4117	-	7,9	0.00	3127	247.2
E6	3927	4543.0	9,8	1.00	3325	587.3
E7	5205	6721.7	10,4	1.00	4763	1233.1
E8	**1051**	1152.9	5,9	1.00	1051	212.1
E9	132	228.7	2,9	1.00	108	0.0
E10	4986	-	10,0	0.00	3400	308.2
E11	4526	5784.5	12,1	1.00	4426	309.5
E12	1010	1200.2	4,1	1.00	380	0.0
E13	173	233.8	5,5	1.00	121	2.0
E14	63	82.3	1,6	1.00	4	1.0
E15	3556	3945.8	13,0	1.00	3362	484.5
M1	5657	6075.0	7,3	0.06	5177	2857.5
M2	5H	-	7,5	0.00	7381	2909.8
M3	**9542**	11403.1	12,4	0.23	9542	3266.8
M4	16	33.0	1,6	1.00	7	7.0
M5	510	624.4	1,7	1.00	413	46.8
M6	1701	2186.3	5,9	1.00	1120	23.0
M7	2203	2452.7	4,5	1.00	1783	23.6

Inst.	Best	Average values cost	time (*h*)	feas.	Best	CPLEX LB
M8	136	196.6	5.5	1.00	129	2.0
M9	640	772.1	4.9	1.00	450	0.0
M10	1357	1687.5	4.0	1.00	1250	3.8
M11	2696	2996.5	12.2	1.00	2446	345.0
M12	950	1054.2	4.1	1.00	911	1.0
M13	362	479.3	4.4	1.00	252	0.0
M14	**1172**	1304.6	10.4	1.00	1172	0.0
M15	985	1099.7	2.4	1.00	485	0.7
L1	2021	2372.7	5.6	1.00	1922	1102.4
L2	5715	6085.5	11.5	0.49	5400	2817.7
L3	2457	2718.0	5.1	1.00	2369	347.9
L4	**0**	0.0	0.7	1.00	0	0.0
L5	2341	-	2.6	0.00	1923	397.5
L6	930	1121.3	2.0	1.00	923	5.4
L7	1765	2226.5	6.4	1.00	1558	3.1
L8	997	1155.3	3.1	1.00	934	77.9
L9	715	881.2	7.2	1.00	563	2.9
L10	2571	3527.3	9.0	0.05	1945	1.0
L11	207	289.3	4.4	1.00	202	0.0
L12	3944	4830.6	9.9	1.00	3428	156.2
L13	1868	2285.5	5.8	1.00	1820	6.1
L14	**1202**	1326.3	10.9	1.00	1202	6.5
L15	60	82.8	1.8	1.00	20	0.0

PartialSwapTeamsPhased - Effectiveness

Instance	With PSTP avg	With PSTP feasible (%)	Without PSTP avg	Without PSTP feasible (%)	gap (%)
Early_1	540.7	1.00	563.5	1.00	+4.22%
Early_2	384.6	1.00	388.3	1.00	+0.96%
Early_3	1176.5	1.00	1204.4	1.00	+2.37%
Early_4	1007.8	0.56	1125.8	0.38	+11.71%
Early_5	-	0.00	-	0.00	-
Early_6	4543.0	1.00	4553.2	1.00	+0.22%
Early_10	-	0.00	-	0.00	-
Early_12	1200.2	1.00	1326.4	1.00	+10.51%
Middle_1	6075.0	0.06	-	0.00	$+\infty$
Middle_2	-	0.00	-	0.00	-
Middle_4	33.0	1.00	33.3	1.00	+0.91%
Middle_5	624.4	1.00	656.8	1.00	+5.19%
Middle_6	2186.3	1.00	2224.7	1.00	+1.76%
Middle_10	1687.5	1.00	1766.8	1.00	+4.70%
Middle_11	2996.5	1.00	3131.8	1.00	+4.52%
Middle_12	1054.2	1.00	1137.0	1.00	+7.85%
Late_4	0.0	1.00	0.0	1.00	+0.00%
Late_5	-	0.00	-	0.00	-
Late_6	1121.3	1.00	1141.7	1.00	+1.82%
Late_8	1155.3	1.00	1186.1	1.00	+2.67%
Late_10	3527.3	0.05	3590.0	0.05	+1.58%
Late_11	289.3	1.00	321.6	1.00	+11.16%

Averages on 20 runs per instance and configuration.

Comparison with our MIP Model

$\lvert\mathcal{T}\rvert$	# Inst.	# Feas.	# Opt.
6	20	7	13
8	20	14	6
10	20	15	2
12	20	4	1
14	20	2	0

Removed constraints	# Inst.	# Feas.	# Opt.
—	45	1	0
CA	45	3	3
GA	42	1	0
BR	45	3	0
FA	23	0	0
SE	24	0	1
CA, GA	42	8	3
CA, BR	45	20	6
CA, FA	23	6	4
CA, SE	24	4	3
GA, BR	42	7	0
GA, FA	22	0	0
GA, SE	24	1	0
BR, FA	23	4	3
BR, SE	24	2	2
FA, SE	3	0	0

Tests realized with 1h time limit

Conclusions and Future Work

- Three-stages multi-neighborhood Simulated Annealing
- PartialSwapTeamsPhased
- 44/45 instances solved to feasibility
- 2nd out of 13 participants at ITC2021
- Future work:
  - Test our SA on different (real-world) instances
  - Feature-based tuning
  - Hybrid approaches (LNS, CMSA)

Questions?

robertomaria.rosati@uniud.it